Correlations and forces in sheared fluids with or without quenching

[New J. Phys. **21** 073029 (2019)]

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TPCE19, Erice, 09. 2019

Collaborators

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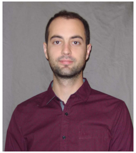
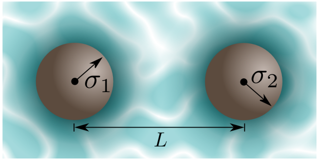


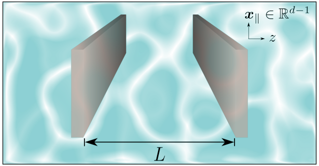
Mehran Kardar
(MIT)

Overview

System

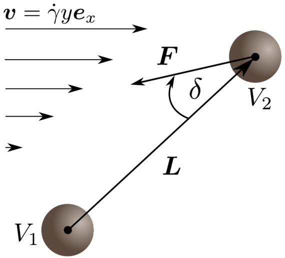
- Stochastic media: classical fluids (no hydrodyn.)
- Short-ranged correlations in equilibrium
- External objects
- Collective phenomena & effects on external objects





Non-equilibrium phenomena

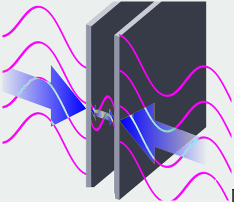
- Temperature quench: dynamics & correlations
- Additionally shear
- Role of conservation laws?



Fluctuation-induced forces

Necessary conditions

- Fluctuations
- Long-ranged correlations
- External bodies modify boundary cond's



[wikipedia]

Review: *Kardar*, *Golestanian*, Rev. Mod. Phys. (1999)

Examples in *equilibrium*:

Quantum Casimir force

$$\frac{F}{A} \propto \frac{\hbar c}{L^4}$$

H.B. Casimir, Proc. K. Ned. Adad. Wet. (1948)

Thermal/Critical Casimir force

$$\frac{F}{A} \propto \frac{k_B T}{L^3}$$

Fisher, *de Gennes*, CR Acad. Sci. Paris. Ser. (1978)

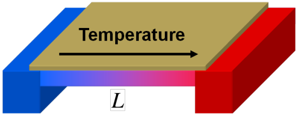
NB: $F \to 0$ if $\xi \to 0$!

Purely non-equilib. FIFs

PRL **110,** 235902 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Giant Casimir Effect in Fluids in Nonequilibrium Steady States

T. R. Kirkpatrick,[1,2,*] J. M. Ortiz de Zárate,[3] and J. V. Sengers[1]

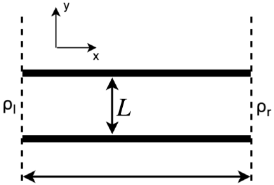


$$p_{\mathrm{NE}} = \frac{c_p k_{\mathrm{B}} T^2(\gamma - 1)}{96\pi D_T(\nu + D_T)} \times \left[1 - \frac{1}{\alpha c_p}\left(\frac{\partial c_p}{\partial T}\right)_p + \frac{1}{\alpha^2}\left(\frac{\partial \alpha}{\partial T}\right)_p\right] L \left(\frac{\nabla T}{T}\right)^2.$$

PRL **114,** 230602 (2015) PHYSICAL REVIEW LETTERS week ending 12 JUNE 2015

Fluctuation-Induced Forces in Nonequilibrium Diffusive Dynamics

Avi Aminov Yariv Kafri Mehran Kardar



$$F = -\frac{k_B T}{L}(\Delta\rho)^2 g(\rho_l, \rho_r)$$

Here: perform quench

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Fluid with local/short-ranged correlations in equilib. (no hydrodyn.)

Perform temperature quench

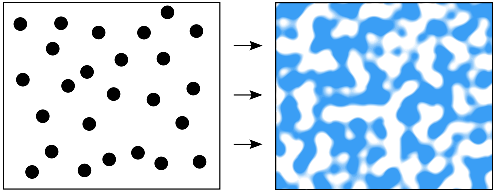
- rapidly switch noise strength
  - passive medium: **temperature** $\Leftrightarrow T$
  - active medium: **activity** $\Leftrightarrow T_{eff}$

Later: add shear to system (inhomogeneous)

- microscopic Langevin eqs

  $d\mathbf{x}_i/dt = \frac{D}{k_B T}\mathbf{F}_i + \sqrt{2D}\boldsymbol{\xi}_i(t)$

  $\langle \boldsymbol{\xi}_i(t)\rangle = 0, \quad \langle \boldsymbol{\xi}_i(t)\boldsymbol{\xi}_j(t')\rangle = \delta_{i,j}\delta(t-t')$



[J. Chem. Phys. **146**, 134507 (2017)]

[J. Chem. Phys. **148**, 084503 (2018)]

- coarse-grain: density field

  $\rho(\mathbf{x},t) = \sum_i \delta(\mathbf{x}-\mathbf{x}_i(t))$

- fluctuations

  $\phi(\mathbf{x},t) = \rho(\mathbf{x},t) - \langle \rho(\mathbf{x})\rangle_{eq}$

- Langevin eq. for ϕ:

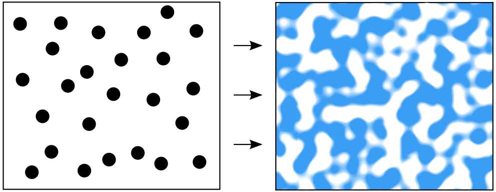
$$\partial_t \phi(\boldsymbol{x},t) \;=\; \mu\nabla^2[m\phi(\boldsymbol{x},t)] + \eta(\boldsymbol{x},t) \;=\; -\nabla\cdot\mathbf{j}$$

$$\langle \eta(\boldsymbol{x},t)\eta(\boldsymbol{x}',t')\rangle = -2k_B T\mu\nabla^2\delta^d(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t')$$

- $\mathscr{H}[\phi] = \int \mathrm{d}^d x\, \frac{m}{2}\phi^2(\mathbf{x})$ [no $(\nabla\phi)^2$-term, i.e. $\xi \to 0$] (!)

Correlations following a quench

- $\phi(\boldsymbol{x},t) = \rho(\boldsymbol{x},t) - \langle \rho(\boldsymbol{x}) \rangle_{eq}$
- $\mathscr{H}[\phi] = \int \mathrm{d}^d x \, \frac{m}{2} \phi^2(\boldsymbol{x})$
- steady state: $\langle \phi(\boldsymbol{x}) \phi(\boldsymbol{x}') \rangle = \delta^d(\boldsymbol{x} - \boldsymbol{x}') \frac{k_B T}{m}$



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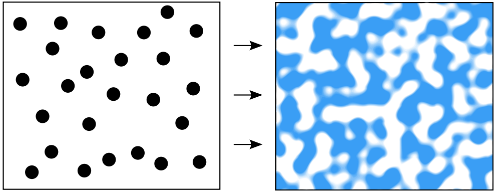

- **Dynamics**

$$\partial_t \phi(\boldsymbol{x},t) = \mu \nabla^2 [m \phi(\boldsymbol{x},t)] + \eta(\boldsymbol{x},t) \quad \text{with} \quad \langle \eta(\boldsymbol{x},t) \eta(\boldsymbol{x}',t') \rangle = -2 k_B T \mu \nabla^2 \delta^d(\boldsymbol{x} - \boldsymbol{x}') \delta(t - t')$$

- **Quench:**
  - Initial state: $\phi(t=0) = 0$ $(T = 0)$
  - At time $t = 0$: **switch on noise** $(T > 0)$
  - System must reach new equilib.

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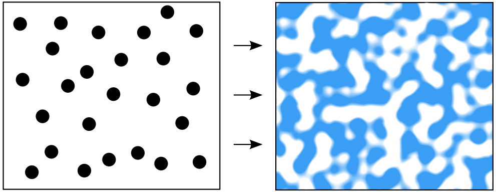

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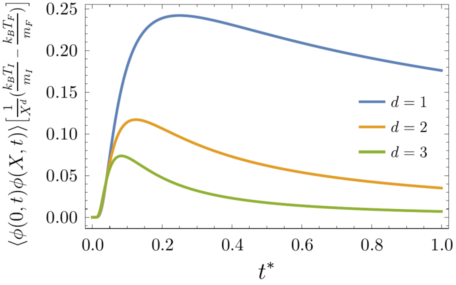
- **Quench:**
  - Initial state: $\phi(t=0) = 0$ $(T=0)$
  - At time $t=0$: **switch on noise** $(T>0)$
  - System must reach new equilib.
- **Transient long-ranged correlations:**

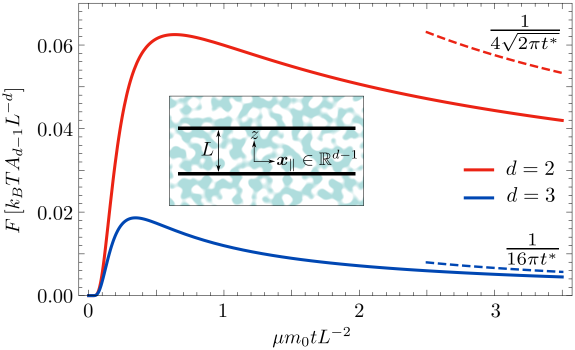
$$\langle \phi(0,t) \phi(X,t) \rangle = -\frac{k_B T}{m X^d} \frac{e^{-\frac{1}{8t^*}}}{(8\pi t^*)^{d/2}},$$

$$t^* = \mu m t / X^2.$$



Transient fluct.-ind. force: plates

[Rohwer *et al.*, Phys. Rev. Lett. **118**, 015702 (2017)]

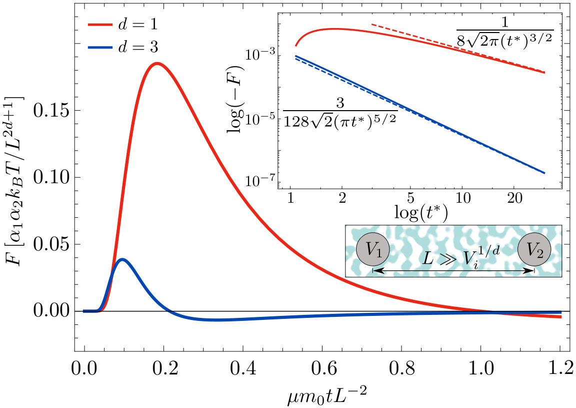


$$\frac{F(t)}{A} = \frac{k_B T}{L^d} \frac{\vartheta_3\left(0, e^{-\frac{1}{2t^*}}\right) - 1}{(8\pi t^*)^{d/2}}$$

- $t^* = \mu m_0 t / L^2$
- $A = \text{area} \in \mathbb{R}^{d-1}$
- $\vartheta_3(0, q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}$

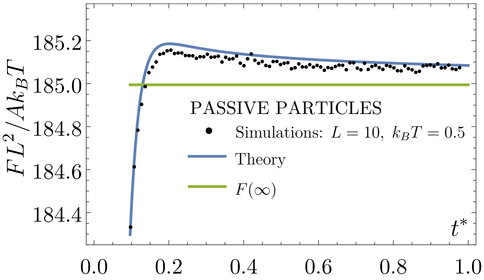
Transient fluct.-ind. force: inclusions

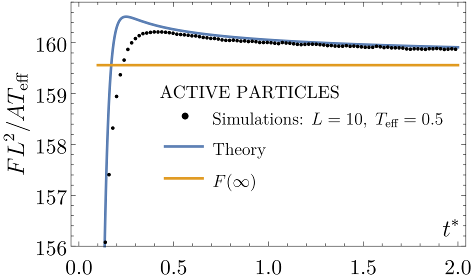
[Rohwer *et al.*,
Phys. Rev. Lett. **118**,
015702 (2017)]



$$\frac{F(t)}{A} = \frac{k_B T}{L^{2d+1}}\alpha_1\alpha_2 e^{-\frac{1}{2t^*}} \begin{cases} \frac{(1-t^*)}{16\sqrt{2\pi}(t^*)^{5/2}}, & d=1, \\ \frac{[1-t^*(3t^*+4)]}{256\sqrt{2}\pi^{5/2}(t^*)^{9/2}}, & d=3. \end{cases}$$

Simulations: interacting particles





Simulations:

- Interacting Brownian particles
- T-quench for passive particles
- activity quench for passive particles

[Rohwer *et al.*, Phys. Rev. E **97**, 032125 (2018)]

What happens if we additionally shear this (non-eq.) system?

Dynamics with shear

Details: New J. Phys. **21** 073029 (2019)

Density fluctuations: $\phi(\boldsymbol{x},t) = \rho(\boldsymbol{x},t) - \langle\rho(\boldsymbol{x})\rangle_{eq}$

Langevin eq.:

$$\partial_t\phi + \dot{\gamma}y\frac{\partial\phi}{\partial x} = \hat{\mu}(\xi^2\nabla^2 - 1)\phi + \eta(\boldsymbol{x},t)$$
$$\langle\eta(\boldsymbol{x},t)\eta(\boldsymbol{x}',t')\rangle = 2\hat{\mu}k_BT\delta(\boldsymbol{x}-\boldsymbol{x}')\delta(t-t').$$

- $\dot{\gamma}$ = shear rate
- $\xi = \sqrt{\kappa/m}$ = equilib. correlation length (e.g. ∵ interaction radius)
- m = mass / compressibility

Conserved / non-conserved dynamics:

$$\hat{\mu} = \begin{cases} \mu_A, & \text{non-conserved,} \\ -\mu_B\nabla^2, & \text{conserved.} \end{cases}$$

Correlations in presence of shear

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Steady state after quench ($t \to \infty$):

- Model A (non-conserved): $\langle \phi(\boldsymbol{x})\phi(\boldsymbol{x}')\rangle_{t\to\infty} \propto \dot{\gamma}\frac{e^{-x/\xi}}{|\boldsymbol{x}|^3}$
- Model B (conserved): $\langle \phi(\boldsymbol{x})\phi(\boldsymbol{x}')\rangle_{t\to\infty} \propto \dot{\gamma}\frac{\xi^2}{|\boldsymbol{x}|^3}$

Correlations in presence of shear

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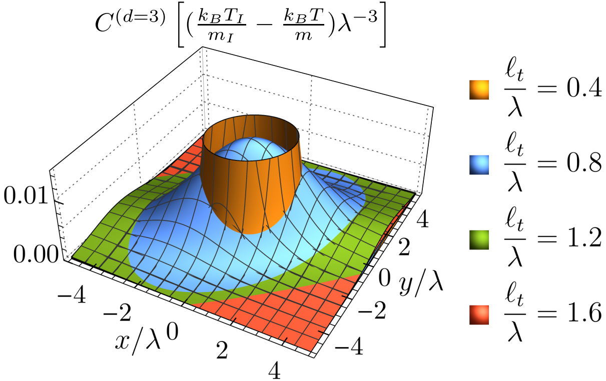
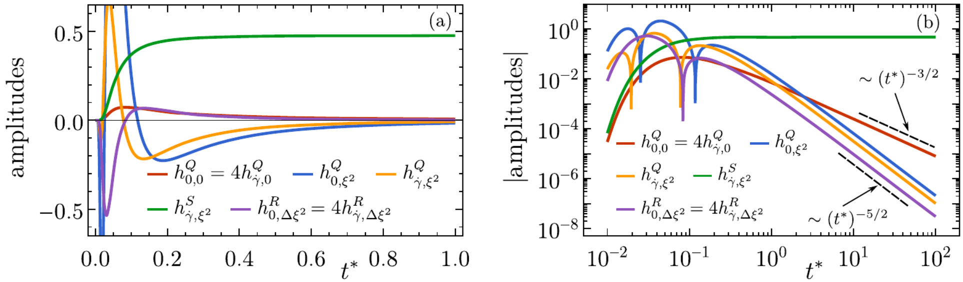
Dynamics:



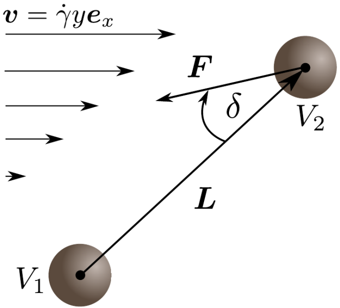
Correlations: conserved dynamics

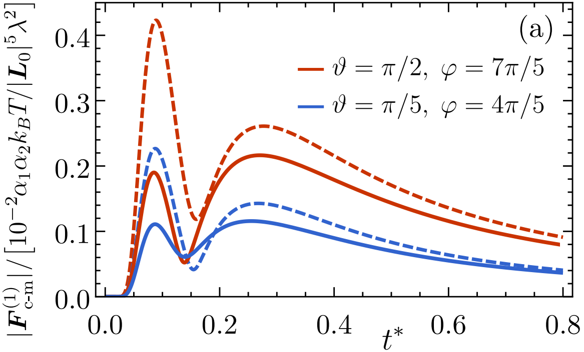
$$C^{(d=3)}(\boldsymbol{x},\,t) = \frac{1}{|\boldsymbol{x}|^3}\left(\frac{k_B T_I}{m_I} - \frac{k_B T}{m}\right)\left[h^Q_{0,0}(t^*) + \tilde{\xi}^2 h^Q_{0,\xi^2}(t^*) + \frac{\Omega_x\Omega_y}{\tilde{\lambda}^2} h^Q_{\dot{\gamma},0}(t^*) + \frac{\Omega_x\Omega_y\tilde{\xi}^2}{\tilde{\lambda}^2} h^Q_{\dot{\gamma},\xi^2}(t^*)\right]$$
$$+ \frac{k_B T}{m|\boldsymbol{x}|^3}\left[\frac{\Omega_x\Omega_y\tilde{\xi}^2}{\tilde{\lambda}^2} h^S_{\dot{\gamma},\xi^2}(t^*)\right] + \frac{k_B T_I}{m_I|\boldsymbol{x}|^3}\Delta\tilde{\xi}^2\left[h^R_{0,\Delta\xi^2}(t^*) + \frac{\Omega_x\Omega_y}{\tilde{\lambda}^2} h^R_{\dot{\gamma},\Delta\xi^2}(t^*)\right]$$
$$+ \mathcal{O}(\tilde{\xi}^4,\,\Delta\xi^4,\,\lambda^{-4}),$$

$\lambda = \sqrt{D/\dot{\gamma}}$, i.e. $\lambda^{-2} \sim \dot{\gamma}$; m = mass/compressibility; $t^* = Dt/x^2$ = dim.less time;
ξ = fluid correlation length; T_I, T = temp. before/after quench; $\Omega_\alpha = (\hat{x})_\alpha$



Fluctuation-induced forces with shear



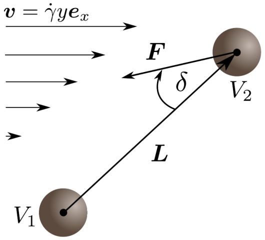


- Magnitude depends on orientation of $\boldsymbol{L}$
- Direction: not along $\boldsymbol{L}$ ($\because$ shear distortion)

Closing

Fluctuation-induced effects in sheared systems

- Quench + conservation $\Longrightarrow$ Long-ranged non-eq. correlations
- Embedded objects: transient forces
- Shear distorts correlations
- Non-trivial time-dep. of forces



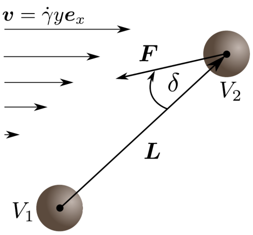
Outlook

- Plate geometry
- $n > 2$ inclusions (many-body)
- Transport coefficients, e.g., viscosity
- Other geometries

Thank you!

Correlations & forces with shear

[1] C.M. Rohwer, A. Maciolek, S. Dietrich & M. Krüger
New J. Phys. **21** 073029 (2019)



Fluctuation-induced forces from T-quenches:

[2] C.M. Rohwer, M. Kardar & M. Krüger
Phys. Rev. Lett. **118**, 015702 (2017)

[3] C.M. Rohwer, A. Solon, M. Kardar & M. Krüger
Phys. Rev. E **97**, 032125 (2018)